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A note on the scaling relations for opening mode fractures in rock

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ABSTRACT

It is well established that shear cracks in rock (faults) obey linear displacement–length scaling and thus have scale invariant driving stresses. Several recent papers have claimed that for opening mode cracks in rock (joints, veins, and dikes) displacement obeys square root scaling with fracture length. This is a fundamentally different mode of behavior, because, unlike shear cracks, opening mode cracks would then be unstable under constant stress boundary conditions. Here the same data are reexamined and it is found, to the contrary, that for opening mode cracks in rock, fracture toughness K_c scales with $\sqrt{L}$ and hence displacement scales linearly with L. The conflicting view resulted from data misinterpretation. This resolves the discrepancy between the behavior of shear and opening mode cracks in rock.

1. Introduction

The basic scaling law for cracks relates the displacement of the crack walls d to crack length L through the stress drop (driving stress) $\Delta\sigma$ with a linear relation. In the case of an elastic crack this is:

$$d_{\max} = \Delta\sigma\frac{2(1-\nu^2)}{E}L. \quad (1)$$

If cracks grow under constant stress loading, we would expect $\Delta\sigma$ to be scale independent and $d_{\max}$ to scale linearly with L. This is indeed the scaling relation found for shear cracks, i.e. faults and earthquakes (for a recent review, see Scholz, 2007). Shear cracks in rock are stable under constant stress loading because fracture energy G_c increases linearly with L, hence there is no Griffith-type instability (Cowie and Scholz, 1992). In contrast, for opening mode (tensile) cracks, i.e., joints, veins, and dikes, Olson (2003) and Schultz et al. (2008a) have argued that $d_{\max}$ scales as $\sqrt{L}$.

In the linear elastic fracture mechanics (LEFM) formulation (e.g., Lawn and Wilshaw, 1975), the criterion for crack propagation is:

$$K_c = \Delta\sigma\sqrt{\frac{L}{2\pi}} \quad (2)$$

where K_c is the fracture toughness, which is often assumed in LEFM to be a material constant. Combining (1) and (2) gives

$$d_{\max} = \frac{K_c(1-\nu^2)}{E\sqrt{\frac{\pi}{8}}}\sqrt{L} \quad (3)$$

so that the $\sqrt{L}$ scaling can be interpreted as meaning that K_c is scale invariant. In such a case, Eq. (2) shows that the equilibrium driving stress of the crack must fall as the crack grows. Such cracks would be unstable under constant stress loading and therefore must be assumed to grow under constant displacement boundary conditions (e.g. Segall, 1984).

These results suggest that tensile and shear cracks behave in fundamentally different ways. Why this should be is very puzzling. The differences between shear and tensile cracks are (a) they have different geometrical terms in their crack-tip stress fields, and (b) the former supports residual friction between its walls whereas the latter does not. Neither of these appear to offer an explanation for this fundamental difference in behavior: the friction of faults, for example, is differenced out in the $\Delta\sigma$ term. This demands a re-examination of the scaling of joints, dikes, and veins proposed by Olson (2003) and Schultz et al. (2008a).

2. Reanalysis of the data

The observations relevant to this problem are shown in Fig. 1 (modified from Schultz et al., 2008a). There $d_{\max}$ is plotted vs. L for various data sets of opening mode fractures. Each data set is fit with a function $d_{\max} = C\sqrt{L}$, where

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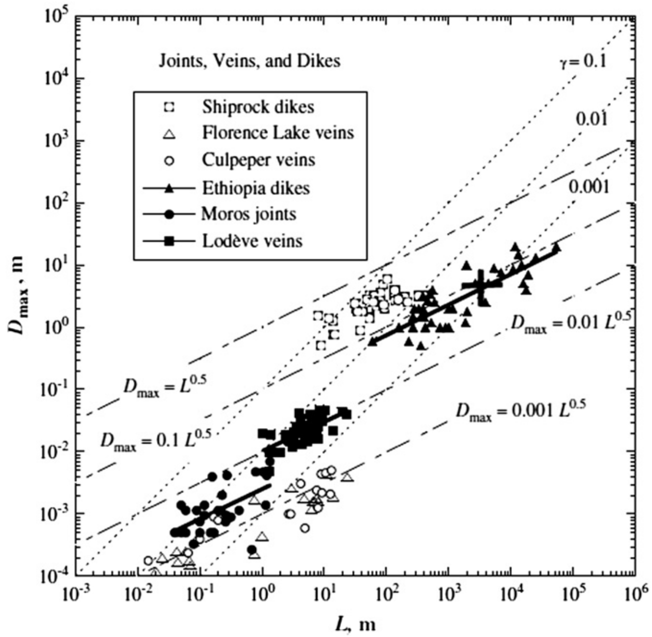

Fig. 1. Compilation of data on joints, veins and dikes, from Schultz et al., 2008a. Fits to $\sqrt{L}$ scaling are in heavy lines. Large cross is Ship Rock master dike.

$$C = \frac{K_c(1-\nu^2)}{E\sqrt{\pi/8}}. \quad (4)$$

In this interpretation the value of C varies by more than three orders of magnitude and tends to progressively increase with the length range of each dataset. This great variation in C cannot be due to variation of the elastic constants, which do not vary by more than about a factor of two or three between different rock types. It must reflect large variations of K_c between data sets.

Olson (2003) recognized this problem, and citing Pollard (1987), noted that the km scale Ship Rock dikes were associated with 10 m wide joint clusters that constitute brittle process zones. Because fracture energy G_c represents the sum of the surface energy expended in creating all the cracks in the process zone, this can be expected to result in a much greater fracture energy G_c and hence K_c for those dikes than for the smaller scale veins and joints. He did not follow this line of inquiry further. Pollard and Segall (1987), discussing the same dike family, derived an expression for the process zone which shows that the process zone size (and hence G_c) scales linearly with L. Available data support this. At the laboratory (cm) scale, Mode I fracture propagation in rock is accompanied by the development of a mm scale process zone consisting of a volumetric region of microcracking surrounding the fracture tip (Peng, 1975; Swanson, 1987). The size of the process zone increases with fracture length, resulting in a corresponding increase of K_c and G_c with L (Labuz et al., 1985, 1987; Peck et al., 1985a,b). Segall and Pollard (1983) noted that the terminations of joints of length 1–10 m in the Sierra Nevada consist of cm scale arrays of sub-parallel cracks, suggesting process zones intermediate in scale to those just described. Engvik et al. (2005, 2009) identified mineralized haloes that surround dikes with their process zones, and showed that they scale linearly with dike displacement.

These observations suggest that K_c scales with L, which contradicts the interpretation shown in Fig. 1. To evaluate this, an average K_c was calculated for each data set in Fig. 1, using Eq. (4), the published C value in Olson (2003) and Schultz et al. (2008a) and appropriate elastic constants for the host rocks. This procedure was not followed for the Ship Rock data set because that consists of echelon segments of a single long dike, which, for reasons given later, are not appropriate for this analysis. In the Ship Rock case K_c was calculated for the entire dike using Eq. (3), with $L = 2900$ m and $d_{max} = 3.9$ m (large cross in Fig. 1; d_{max} is the corrected value from Delaney and Pollard (1981)) and an average value of E for the shale host rock of 19 GPa (Birch, 1966). The results are given vs. the logarithmic mean L for each data set in Table 1 and Fig. 2.

Table 1
Calculations of K_c.

Dataset	C, $M^{1/2}$	E,[a] GPa	L, Log-med, m	K_c, MPa $m^{1/2}$
Culpepper	6.2×10^{-4}	20	0.7	7.6
Florence Lake	6.8×10^{-4}	40	0.8	16.8
Ethiopia	8.8×10^{-2} [b]	73	2000	3682
Moros	2.5×10^{-3}	20	0.3	31
Ledeve	1×10^{-2}	20	5	124
Ship Rock	–	19	2900	850

[a] Elastic moduli average low pressure values from Birch (1966).
[b] From Schultz et al. (2008b).

Fig. 2 makes clear that K_c scales with L. The fit to the data yields $K_c = 26.8L^{.54}$, $R^2 = 0.87$. It thus appears that $K_c \propto \sqrt{L}$. Inserting this into Eq. (3) indicates that $d_{max} \propto L$, which implies that the correct interpretation of the data in Fig. 1 is scaling along the dotted lines of constant d_{max}/L rather than the solid square root lines. From that we see that most of the data lie in the range $d_{max}/L = 10^{-2}$–10^{-3}.

These results are consistent with the earlier findings of Vermilye and Scholz (1995), who reported linear scaling for veins and dikes over about seven orders of magnitude in length scale, with d_{max}/L ratios in the range 10^{-2}–10^{-3}. They also found that the d_{max}/L ratios of segmented veins were systematically much smaller than those of isolated single segment veins. They attributed this difference to the effect of elastic stress interactions between segments, following the analysis of Pollard et al. (1982).

Olson (2003) reinterpreted the Vermilye and Scholz data for two localities: Culpepper Quarry and Florence Lake. He lumped together the single and multiple segment veins in both cases and fitted each to a single curve with exponent ~ 1/2. Examination of his Fig. 5 shows that the single segment and multiple segment data in each case occupy different populations, with the multiple segment veins having distinctly smaller aspect ratios (these two populations of

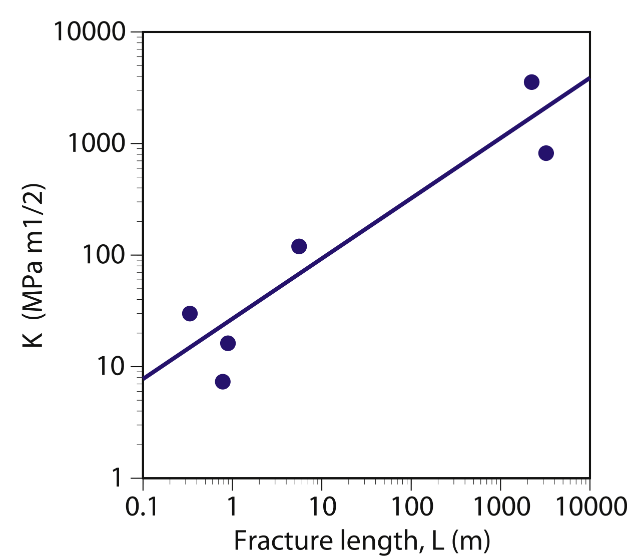

Fig. 2. Calculated values of K_c plotted vs. logarithmic median length for each dataset in Fig. 1. K_c was calculated from Eq. (4) using values of C given by Olson (2003) and Schultz et al. (2008a,b) except for the Ship Rock dike, which was calculated using Eq. (3).

different aspect ratio are also clear in his Fig. 4, although there he did not distinguish in the figure between single and multiple segment veins). Because the longer veins are the multiple segment ones, this conspires to skew the fit to have a lower exponent. Vermilye and Scholz emphasized that the multiple segment veins are not self-similar to the single segment ones (see their Fig. 9). They are thus qualitatively different objects and one cannot define a scaling law that includes both.

In his second case, Olson analyzed individual segments of a long dike near Ship Rock, New Mexico studied by Delaney and Pollard (1981). The latter interpreted these segments as being near surface echelon twist bifurcations (lances) resulting from mixed mode I + III fracture propagation – an interpretation later studied numerically by Pollard et al. (1982) and experimentally by Cooke and Pollard (1996). This interpretation and the original data suggest that the d_{max} of each segment simply reflects the local d of the underlying master dike. On the other hand, the segment length is determined by an instability in mixed mode fracture propagation with spacing proportional the ratio K_I/K_{III} times the process zone size (Pons and Karma, 2010) and which is primarily a function of the twist angle (Lin et al., 2010). Thus in this case d_{max} and L are independent of one another and there should not be any meaningful scaling law relating them (in trying to do so, Olson obtained a fit with a poor correlation coefficient of $R^2 = 0.55$). This is the reason we did not analyze these data but only the data for the master dike. This also explains why these data are an outlier population in Fig. 1. They should not be interpreted with Eq. (2) or (3).

Thus, the case made by Olson (2003) for $\sqrt{L}$ scaling was based on misinterpretations of the data and the apparent goodness of fit of the solid lines in Fig. 1 is illusory. Schultz et al. (2008a) added some new data sets: the only one with a broad enough length span to be discussed is that of the Ethiopian dikes, which Schultz et al. (2008b) fit with an exponent 0.48 and $R^2 = 0.66$. These were multi-segmented dikes, which brings up another problem. Vermilye and Scholz (1995) noted that their segmented veins fit a square root curve marginally as well as a linear one, and attributed this to interactions between segments as shown in the numerical analysis of Pollard et al. (1982). Olson (2003) analyzed this effect in more detail. He showed that the effect of segment interaction is to produce a non-linear reduction of aperture that increases with length – the effect being greater with greater interaction (which is a function of segment overlap and separation). He noted that if one were to fit this with a power law, as the interaction increases it would yield exponents e progressively smaller than one. He stated: "The wide range in values for e suggests that there is no unique power law exponent that describes multi-segment aperture-to-length scaling." The fit to the Ethiopian dikes is just such a fit, and as such does not impact our conclusions on the scaling on non-interacting opening mode cracks.

3. Conclusions

In conclusion, for opening mode fractures, $K_c \propto \sqrt{L}$, $d_{max} \propto L$, and as a consequence $G_c \propto L$. These results are in agreement with the argument of Cowie and Scholz (1992) to explain the linear scaling of d_{max} with L for faults. In the framework of the Dugdale–Barenblatt model, they argued that the breakdown zone length s scales with L, hence fracture energy G_c also scales linearly with L. Because $G_c = K_c^2/E$, K_c scales with $\sqrt{L}$. The proportionality of the fault process zone with length was verified by Vermilye and Scholz (1998). The linear scaling between halo width and dike aperture observed by Engvik et al. (2005) is consistent with this.

Opening and shear mode cracks in rock thus behave in the same manner. The primary difference is that the d_{max}/L ratio and hence $\Delta\sigma$ for shear cracks is about 10 times greater than for tension cracks. A simple manifestation of this is that rock is about 10 times stronger in compression than tension.

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